

Supersymmetric Vacuum Configurations in String Cosmology

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We examine in a cosmological context the conditions for unbroken supersymmetry in $N = 1$ supergravity in $D = 10$ dimensions. We show that the cosmological solutions of the equations of motion obtained considering only the bosonic sector correspond to vacuum states with spontaneous supersymmetry breaking. With a non vanishing gravitino-dilatino condensate we find a solution of the equations of motion that satisfies necessary conditions for unbroken supersymmetry and that smoothly interpolates between Minkowski space and DeSitter space with a linearly growing dilaton, thus providing a possible example of a supersymmetric and non-singular pre-big-bang cosmology.

Supersymmetric vacuum states in string theory have been searched restricting to time-independent fields in the low-energy effective action. The result obtained in this case is well known [1]: looking for a vacuum state of the form $T^4 \times K$, where T^4 is a maximally symmetric four dimensional space and K a compact six manifold, one finds that T^4 is necessarily Minkowski space, and requiring $N = 1$ supersymmetry in $D = 4$ space-time dimensions, K is found to be a manifold of $SU(3)$ holonomy. In this Letter we address some aspects of the problem in a cosmological context, i.e. including a time dependence in the metric, dilaton field, etc., and we will compare our results with the ‘pre-big-bang’ cosmological model proposed in ref. [2], which is based on the bosonic part of the string effective action.

We start from the action of $N = 1$ supergravity in $D = 10$. We perform some simple field redefinitions on the action presented in ref. [3] in order to bring it into the so-called string frame, where it reads*

$$S = \frac{1}{2\kappa^2} \int d^{10}x \sqrt{-g} e^{-\phi} \left[R + (\partial\phi)^2 - \frac{1}{12} H_{MNP} H^{MNP} - \bar{\psi}_M \gamma^{MNP} D_N \psi_P - \bar{\lambda} \gamma^M D_M \lambda - \frac{1}{2\sqrt{2}} (\partial_N \phi) \bar{\psi}_M \gamma^N \gamma^M \lambda \right. \\ \left. + (\partial_N \phi) \bar{\psi}^N \gamma^M \psi_M - \frac{1}{32} (\bar{\lambda} \gamma^{ABC} \lambda) \left(\frac{1}{12} \bar{\psi}^M \gamma_{ABC} \psi_M + \frac{1}{2} \bar{\psi}^M \gamma_{MAB} \psi_C - \bar{\psi}_A \gamma_B \psi_C \right) + \dots \right]. \quad (1)$$

The dots in eq. (1) are terms of the type $(H_{MNP} \times \text{fermion bilinears})$, terms with three gravitino fields and one dilatino, and terms with four gravitinos. Their explicit form is not needed below and can be obtained from ref. [3].

If $|\Omega\rangle$ is a vacuum state annihilated by a supersymmetry generator Q , and $\delta\Psi$ is the supersymmetry variation of any fermionic field Ψ , then $\langle\delta\Psi\rangle \equiv \langle\Omega|\delta\Psi|\Omega\rangle = \langle\Omega|\{Q, \Psi\}|\Omega\rangle = 0$. The variation of the bosonic fields give fermionic fields, and their expectation values are automatically zero. So, in our case we need to impose the conditions $\langle\delta\lambda\rangle = \langle\delta\psi_M\rangle = 0$. We consider first the case in which the expectation values of all bilinears in the Fermi fields are set to zero. This corresponds to solutions of the equations of motions of the bosonic part of the action (1), and therefore to the pre-big-bang cosmology studied in [2]. The supersymmetry variations of the dilatino and gravitino field can be found, e.g., in ref. [3]. Writing them in the string frame, and setting the fermion condensates to zero, the equations $\langle\delta\lambda\rangle = \langle\delta\psi_M\rangle = 0$ give

$$\left(\gamma^M \partial_M \phi - \frac{1}{6} H_{MNP} \gamma^{MNP} \right) \eta = 0 \quad (2)$$

$$D_M \eta - \frac{1}{8} H_M \eta = 0, \quad (3)$$

*Our notations are as follows: ϕ is the dilaton field, H_{MNP} is the field strength of the two-form field B_{MN} , the graviton ψ_M is a left-handed Weyl-Majorana spinor, the dilatino λ is a right-handed Weyl-Majorana spinor, and the covariant derivative D_M is with respect to the spin connection $\omega(e)$, which is independent of the fermionic fields [4,3]. Indices A, B, M, N take values $0, \dots, 9$. We use the signature $\eta_{MN} = (-, +, +, \dots, +)$. The conventions for the Riemann and Ricci tensors are $R^M_{NRS} = \partial_R \Gamma^M_{NS} - \dots$, $R_{NS} = R^M_{NMS}$; Γ^M_{NS} is the Christoffel symbol while $\gamma^{ABC\dots}$ denotes the antisymmetrized product of ten-dimensional gamma matrices, with weight one.

where η is the parameter of the supersymmetry transformation and $H_M \equiv H_{MNP}\gamma^{NP}$. Note that in the string frame eq. (3) is independent of the dilaton field, contrarily to what happens in the Einstein frame [1]. This simplifies considerably the analysis of the equations. Writing $\hat{D}_M \equiv D_M - (1/8)H_M$, eq. (3) implies the integrability conditions $[\hat{D}_M, \hat{D}_N]\eta = 0$, which gives

$$[2R_{MNPQ}\gamma^{PQ} + (D_N H_M) - (D_M H_N) - H_M^R{}_Q H_{NRS}\gamma^{QS}]\eta = 0, \quad (4)$$

which is therefore a necessary (but not sufficient) condition for supersymmetry. One can now see by inspection that the solutions used in homogeneous pre-big-bang cosmology [2] do not satisfy equations (2) and (4). This is obvious for the solutions with vanishing H_{MNP} , since in this case eq. (2) requires a constant dilaton. In fact, we also tried a rather general ansatz compatible with a maximally symmetric three-dimensional space, $ds^2 = -dt^2 + a^2(t)d\vec{x}^2 + g_{mn}(t, \vec{y})dy^m dy^n$, in which the 3-space, with coordinates \vec{x} , is isotropic and has a scale factor independent of the internal coordinates \vec{y} , while the metric in the six-dimensional internal space is independent of the x_i but otherwise arbitrary. For H_{MNP} we made the ansatz $H_{ijk} = \text{const.}\epsilon_{ijk}$ for $i, j, k = 1, 2, 3$, $H_{0ij} = 0$, H_{MNP} vanishes also if indices of the three-space and indices of the internal space appear simultaneously, and H_{MNP} is arbitrary if all the indices MNP take values $0, 4, \dots, 9$. We also considered the case of spatially curved sections of the three-space. Even with this ansatz, which is the most general compatible with maximal symmetry of the three-space when the metric of the three-space is independent of the internal coordinates, it is straightforward to show that eqs. (2,4) do not admit non-trivial cosmological solutions.

The super-inflationary pre-big-bang solutions of ref. [2] are therefore rotated by supersymmetry transformations into different classical solutions of the action (1). Since each classical solution of the equations of motion corresponds to a string vacuum, this means that selecting such a vacuum corresponds to a spontaneous breaking of supersymmetry. If we want to preserve the advantages of supersymmetry for low-energy physics, for instance for the hierarchy problem, we do not wish to break supersymmetry already in the pre-big-bang era. Therefore we now look for vacuum states with unbroken supersymmetry. The above results suggest that, in order to find supersymmetric solutions, the effect of Fermi fields must be switched on, which means that we must consider the effect of non-vanishing fermion condensates. We have found a particularly simple and appealing solution assuming that the only non-vanishing fermion bilinear is the mixed gravitino-dilatino condensate, $\langle \bar{\lambda}\psi_M \rangle$. Let us define $v_M = -(\sqrt{2}/8)\langle \bar{\lambda}\psi_M \rangle$. It is a composite vector field and in general in a cosmological setting it depends on time (note that while in global supersymmetry the fermion condensates are space-time independent [5], this is not the case with local supersymmetry). Furthermore, we look for solutions with $H_{MNP} = 0$. In this case the equations $\langle \delta\lambda \rangle = 0$, $\langle \delta\psi_M \rangle = 0$ give

$$\gamma^M(\partial_M \phi - 8v_M)\eta = 0, \quad (5)$$

$$D_M \eta - \left(8v_M + \frac{1}{2}\gamma_M^N v_N\right)\eta = 0. \quad (6)$$

The integrability condition of eq. (6) is

$$[R_{MN}{}^{PQ}\gamma_{PQ} - 2v_A v_B (g^{AB}\gamma_{MN} + \delta_N^A \gamma_M^B - \delta_M^B \gamma_N^A) - 32f_{MN} + 2(\gamma_M^A D_N v_A - \gamma_N^A D_M v_A)]\eta = 0, \quad (7)$$

where $f_{MN} = \partial_M v_N - \partial_N v_M$ and $D_M v_A = \partial_M v_A - \Gamma_{MA}^B v_B$. For the metric we make an isotropic ansatz, $ds^2 = -dt^2 + a^2(t)dx_i dx^i$, with $i = 1, \dots, 9$, and we define as usual the Hubble parameter $H(t) = \dot{a}/a$. Our strategy is to find a field configuration $\phi(t), H(t), v_M(t)$ such that eqs. (5) and (7) are identically satisfied, without requiring any condition on η . This is because eq. (7) is only the integrability condition for eq. (6), and as such it is a necessary but not sufficient condition for unbroken supersymmetry. If it is satisfied for any η we still have the freedom to choose η so that also eq. (6) is satisfied.

Examining eqs. (5) and (7) we see that this is possible only if $v_i(t) = 0$, $i = 1, \dots, 9$. Denoting $v_{M=0}(t) \equiv \sigma(t)$, eq. (5) becomes simply $\dot{\phi} = 8\sigma$. Eq. (7), for $M = 0, N = i$, becomes $\dot{H} - \dot{\sigma} + H(H - \sigma) = 0$, while for $M = i, N = j$ we get $(H - \sigma)^2 = 0$. We see that these equations are identically satisfied by $H(t) = \sigma(t)$. We now ask whether $H(t) = \sigma(t)$, $\dot{\phi}(t) = 8\sigma(t)$ is a solution of the equations of motions, as we expect for a supersymmetric configuration. The equations of motion obtained with a variation with respect to fermionic fields are automatically satisfied when we take the expectation value over the vacuum, and we only need to check the variation with respect to bosonic fields. We introduce the shifted dilaton $\bar{\phi} = \phi - d\beta$, where $\beta = \log a$ and $d = 9$ is the number of spatial dimensions, and we also retain the lapse function N in the metric, so that $ds^2 = -N^2 dt^2 + e^{2\beta} dx_i dx^i$. Restricting to homogeneous fields, the relevant part of the action can be written as

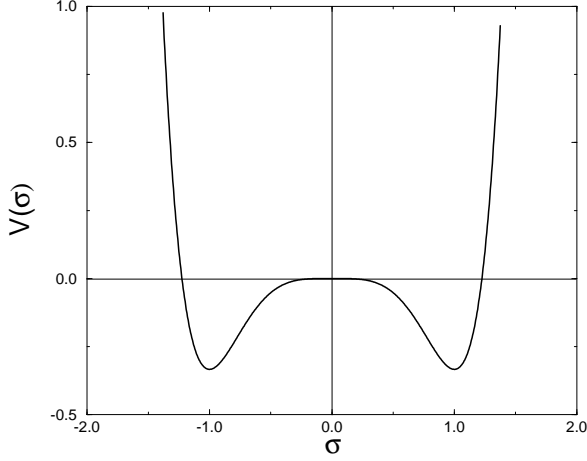


FIG. 1. A symmetry breaking potential for the field $\sigma(t)$.

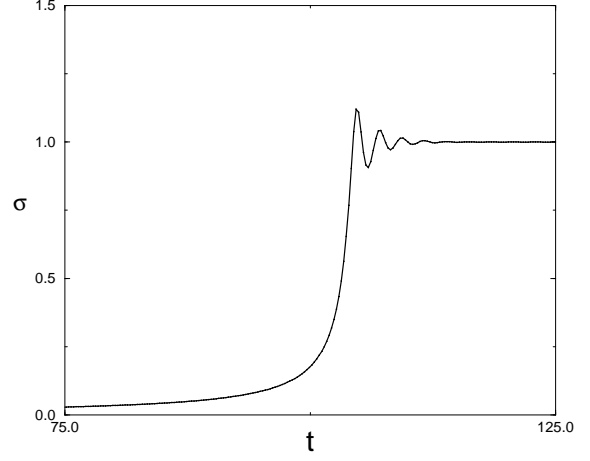


FIG. 2. The evolution of the field $\sigma(t)$.

$$S = -\frac{1}{2\kappa^2} \int dt e^{-\bar{\phi}} \frac{1}{N} \left[-d\dot{\beta}^2 + \dot{\bar{\phi}}^2 + 2 \left(-\frac{\sqrt{2}}{8} \bar{\lambda} \psi_0 \right) \dot{\bar{\phi}} + 2d \left(-\frac{\sqrt{2}}{8} \bar{\lambda} \psi_0 \right) \dot{\beta} - 8 \left(\frac{\sqrt{2}}{8} \bar{\lambda} \psi_0 \right)^2 \right]. \quad (8)$$

The last term in the action (8) comes from the term $(\bar{\lambda} \gamma^{ABC} \lambda)(\bar{\psi}^M \gamma_{ABC} \psi_M)$ in eq. (1), making use of the Fierz identity $(\bar{\lambda} \gamma^{ABC} \lambda)(\bar{\psi}^M \gamma_{ABC} \psi_M) = 96(\bar{\lambda} \psi^M)(\bar{\lambda} \psi_M)$ (see e.g. the appendix of ref. [3]). Instead, the terms $(\bar{\lambda} \gamma^{ABC} \lambda)(\bar{\psi}^M \gamma_{MAB} \psi_C)$ and $(\bar{\lambda} \gamma^{ABC} \lambda)(\bar{\psi}_A \gamma_B \psi_C)$ in the action (1) are independent from $(\bar{\lambda} \psi^M)(\bar{\lambda} \psi_M)$ and their condensates can be consistently set to zero. We now vary the action with respect to $N, \bar{\phi}, \beta$ and then take the expectation value of the terms $\bar{\lambda} \psi_0, (\bar{\lambda} \psi_0)^2$ over the vacuum. We get the equations

$$\frac{d}{dt} \left(e^{-\bar{\phi}} (H - \sigma) \right) = 0, \quad \dot{\bar{\phi}}^2 - 9H^2 + 2\sigma \dot{\bar{\phi}} + 18\sigma H - 8 \left\langle \left(\frac{\sqrt{2}}{8} \bar{\lambda} \psi_0 \right)^2 \right\rangle = 0 \quad (9)$$

$$2(\ddot{\bar{\phi}} + \dot{\sigma}) - 2\dot{\bar{\phi}}(\dot{\bar{\phi}} + \sigma) - 9H^2 + \dot{\bar{\phi}}^2 + 2\sigma \dot{\bar{\phi}} + 18\sigma H - 8 \left\langle \left(\frac{\sqrt{2}}{8} \bar{\lambda} \psi_0 \right)^2 \right\rangle = 0. \quad (10)$$

For the configuration $H = \sigma, \dot{\bar{\phi}} = 8\sigma$ (and therefore $\ddot{\bar{\phi}} = \dot{\bar{\phi}} - 9H = -\sigma$) we see that the equations of motion are identically satisfied if $\langle (\bar{\lambda} \psi_0)^2 \rangle = \langle \bar{\lambda} \psi_0 \rangle^2$. Consistency therefore requires that supersymmetry enforces this relation between the condensates. In general, it is well known that relations of this kind are indeed enforced by supersymmetry; for instance, the relation $|\langle \bar{\chi} \chi \rangle|^2 = \langle \bar{\chi} \chi |^2$ holds for the gaugino condensate in the case of super-Yang-Mills theories [5] and in supergravity coupled to super-Yang-Mills [6].

It remains to discuss the dynamics of the condensate $\sigma(t)$. This is a composite field whose dynamics will be governed by an effective action which in principle follows from the fundamental action (1). To assume that a condensate forms is the same as assuming that the field $\sigma(t)$ has an effective action with a potential $V(\sigma)$ with the absolute minimum at $\sigma = \bar{\sigma} \neq 0$, see fig. 1. If we choose as initial condition $\sigma \rightarrow 0^+$ as $t \rightarrow -\infty$, $\sigma(t)$ will evolve from $\sigma = 0$ toward the positive minimum of the potential, and it will make damped oscillations around $\sigma = \bar{\sigma}$, the damping mechanism being provided by the expansion of the Universe and possibly by the creation of particles coupled to the σ field. The qualitative behaviour of $\sigma(t)$ will be therefore of the form plotted in fig. 2. For illustrative purposes, we have shown in fig. 2 the evolution of σ obtained assuming an effective action, in the string frame, of the form

$$S = \int dt e^{-\bar{\phi}} \left(\frac{1}{2} \dot{\sigma}^2 - V(\sigma) \right) \quad (11)$$

where $V(\sigma) = -\sigma^4 + (2/3)\sigma^6$ is the potential shown in fig. 1. (We use units such that the minimum is at $\bar{\sigma} = 1$.) This gives the equation of motion $\ddot{\sigma} - \dot{\bar{\phi}} \dot{\sigma} + V' = 0$, with $-\dot{\bar{\phi}} = \sigma$ providing the friction term. However, the qualitative behaviour is independent from these specific choices.

Since $H(t) = \sigma(t)$ and $\dot{\phi} = 8\sigma(t)$, this solution corresponds to a cosmological model that starts at $t \rightarrow -\infty$ from Minkowski space with constant dilaton and vanishing fermion condensates, i.e. from the string perturbative vacuum, and evolves toward a De Sitter metric $H = \text{const.}$, with linearly growing dilaton. This is similar to the scenario found in ref. [7] in the bosonic sector with α' corrections. In the present case the scale at which the curvature is regularized is given by the fermion condensate $\bar{\sigma}$ while in [7] it was given by the α' corrections. However, in the case studied in ref. [7], higher order α' corrections were not under control, so that a definite statement about the effectiveness of the regularization mechanism could not be made (see also the discussion in ref. [10]). In the present case, instead, the fact that σ , and therefore H and $\dot{\phi}$, stops growing follows from the general requirement that the potential $V(\sigma)$ be bounded from below and has a minimum, as we expect for the effective potential derived from any well-defined fundamental action as the action (1). As in the case studied in [7], the DeSitter solution should finally be matched to a standard radiation-dominated era. For the matching, $O(e^{\phi})$ corrections to the string effective action are probably important [8], since $\dot{\phi}$ is positive and therefore at some stage e^{ϕ} becomes large. The so-called graceful exit problem however takes a different form in our scenario, since $\dot{\phi} = -\sigma$ is always negative in our model and no ‘branch change’ [9] needs to occur. Instead, when the gauge coupling $\sim e^{\phi}$ becomes strong, gaugino condensation is also expected to occur, suggesting that the gaugino condensate might play a role in matching the De Sitter phase to a radiation dominated era.

It is also interesting to observe that, if at small σ the potential $V(\sigma)$ behaves as $V(\sigma) \simeq -\sigma^4/(2c^2)$, with c a positive constant, then at large negative values of time the solution of the equation of motion for σ is $\sigma(t) \simeq c/(-t)$ and therefore $H(t) \simeq c/(-t)$, corresponding to a super-inflationary stage of expansion, as it also happens for the solutions found in [2] in the bosonic sector of the model.

We conclude stressing that the solution that we have presented has more an illustrative rather than a realistic value. For one thing, we have discussed an isotropic ten-dimensional solution and we have not touched upon the issue of compactification of the extra dimensions. To obtain an anisotropic cosmological model, in which three spatial dimensions expand and six get compactified, most probably one must switch on the effect of the three-form H_{ABC} [11] or include the effect of a dilatino condensate $\bar{\lambda}\gamma_{ABC}\lambda$, or a gravitino condensate such as $\bar{\psi}_A\gamma_B\psi_C$, or a gaugino condensate $\bar{\chi}\gamma_{ABC}\chi$. They can separate 3 spatial dimensions from the remaining six if either all the three indices A, B, C belong to the three dimensional space, or if they are the holomorphic indices of a six-dimensional complex manifold. Still, the toy solution that we have discussed illustrates the role of fermion condensates in a supersymmetric cosmology and provides a novel mechanism for the regularization of the singularity of the pre-big-bang cosmological solutions.

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